DP Analysis & Approaches HL

Maths with Lemon

Lesson 1.6HL: Generalization of the Binomial Theorem

Mathematician: _____

AHL 1.10	Guidance, clarification and syllabus links
Generalized binomial theorem	Extension to fractional and negative indices, $(a+b)^n, \qquad n \in \mathbb{Q}.$ Link to power series expansions (AHL 5.19). Proof is not required.

Warm Up: Counting

The integers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are placed on nine cards. Four cards are selected.

a) Find the number of selections if the largest integer selected is either 5, 6 or 7. **[3]**

b) Find the number of selections if at least two of the four integers selected are even. **[4]**

Infinite Geometric Series

Complete:

Convergence condition: _____ $S_\infty =$ _____.

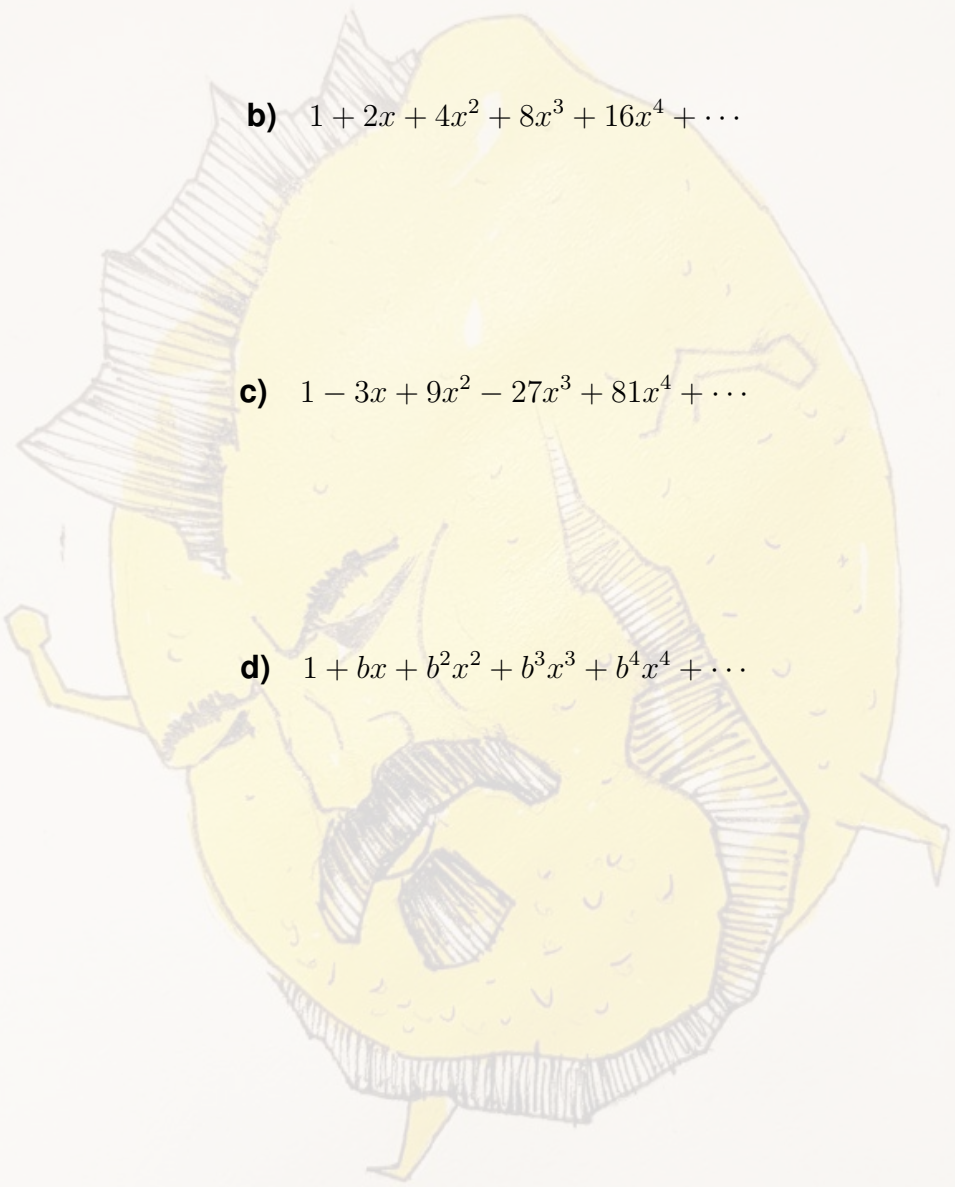
For each series, identify r, determine where it converges, and write its sum.

a) $1 + x + x^2 + x^3 + x^4 + \cdots$

b) $1 + 2x + 4x^2 + 8x^3 + 16x^4 + \cdots$

c) $1 - 3x + 9x^2 - 27x^3 + 81x^4 + \cdots$

d) $1 + bx + b^2x^2 + b^3x^3 + b^4x^4 + \cdots$



The Expansion of $(1-bx)^{-1}$

Complete:

$$(1-bx)^{-1} = \square$$

and

$\square$

is the interval/condition of convergence.

Example

Write the first four terms of

$$(1-4x)^{-1}$$

and state the interval of convergence.

Generalized Binomial Expansion

Complete:

$$(1+x)^n = 1 + \square x + \square x^2 + \square x^3 + \cdots$$

and state its validity condition:

$\square$.

Find the expansion of

$$(1+3y)^{-2}$$

up to and including y^3, and state the values of y for which it is valid.

You Try

Write the first four terms of

$$\left(1+\frac{x}{2}\right)^{-1/2}$$

and state its interval of validity.

IB Formula Booklet

$$(a+b)^n = a^n\left(1+n\frac{b}{a}+\frac{n(n-1)}{2!}\left(\frac{b}{a}\right)^2+\cdots\right)$$

What convergence condition must be satisfied?

Practice

For each, find the **first three terms** and state the values of x for which the expansion is valid.

1. $(3+x)^{-1}$

2. $(8+x)^{1/3}$

3. $(2-3x)^{-3}$

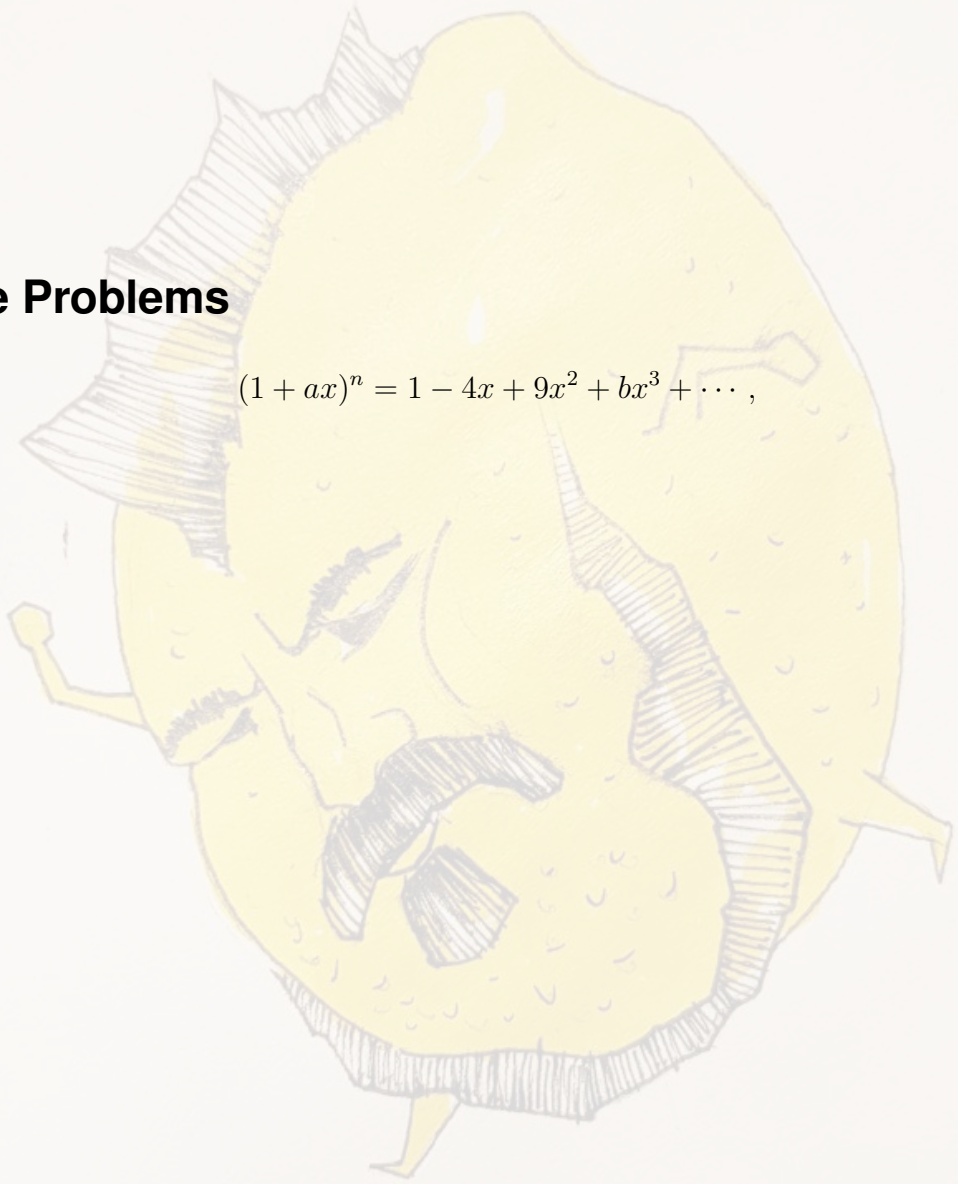
4. $\left(16+\frac{x}{2}\right)^{7/4}$

5. $(5+x)^{-2}$

6. $(9+x)^{1/2}$

7. $(3+4x)^{-1}$

8. $\left(4-\frac{x}{3}\right)^{3/2}$



Challenge Problems

1. Given

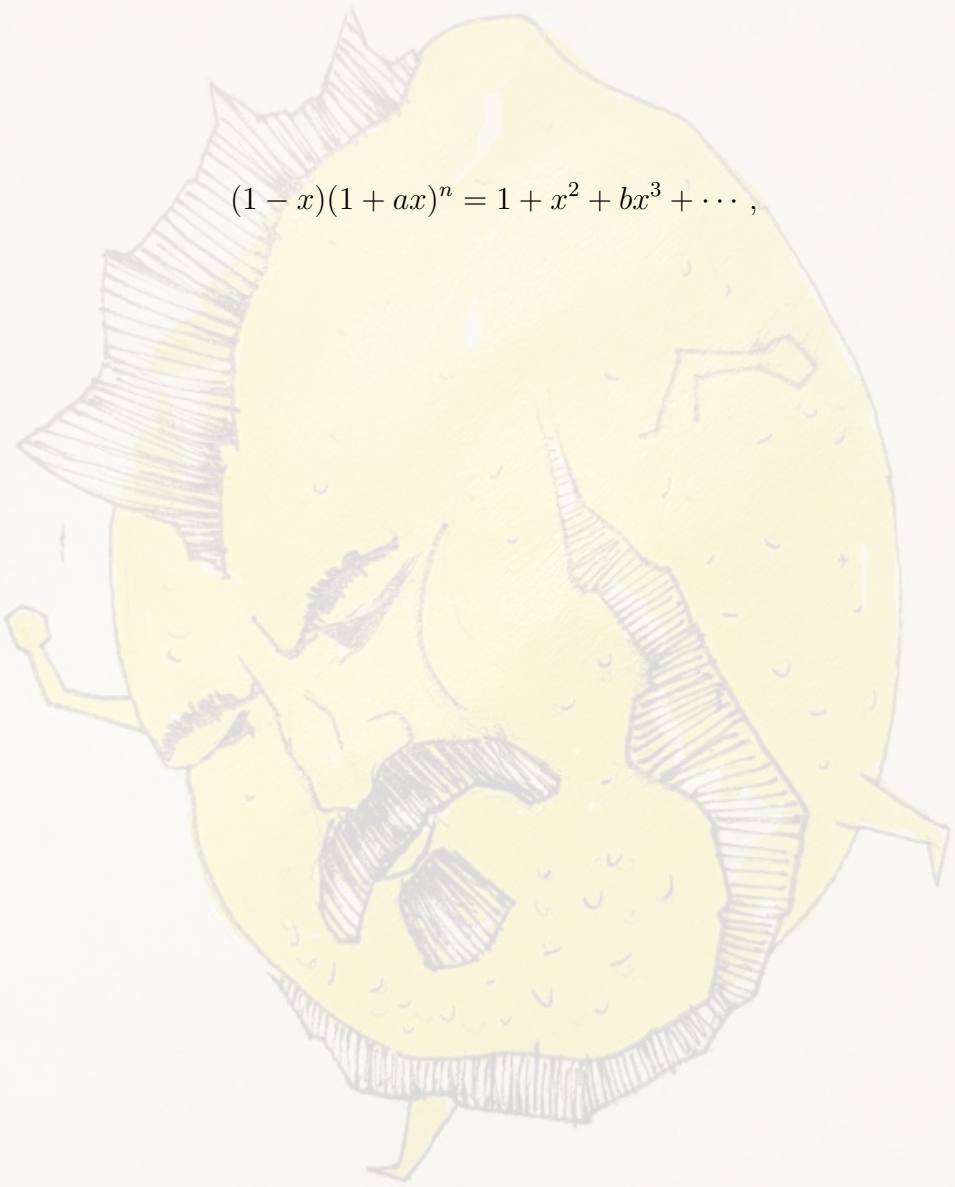
$$(1+ax)^n = 1 - 4x + 9x^2 + bx^3 + \cdots,$$

find b.

2. Given

$$(1+ax)^n = 1 + 21x + bx^2 + bx^3 + \cdots, \qquad b \neq 0,$$

find b.



3. Given

$$(1-x)(1+ax)^n = 1 + x^2 + bx^3 + \cdots,$$

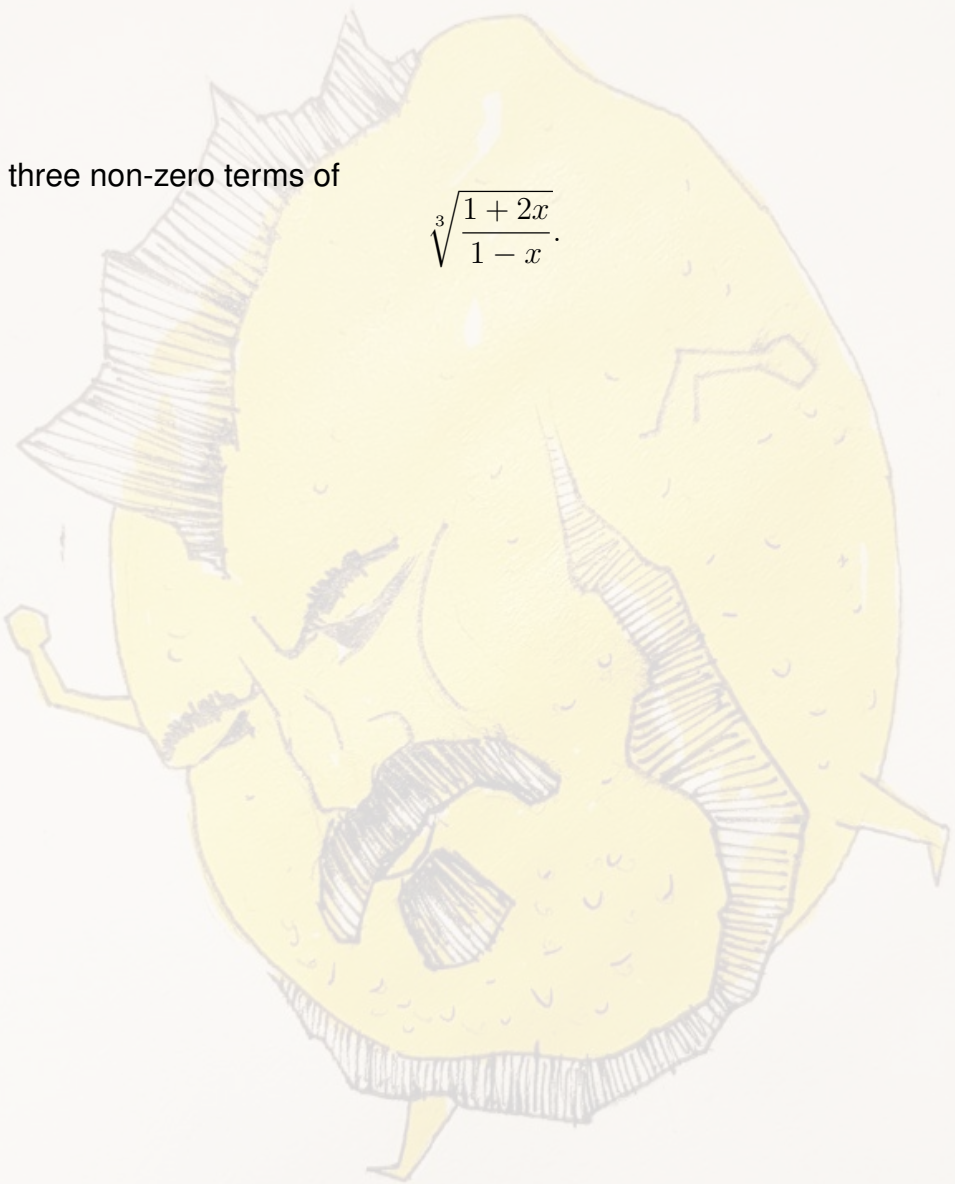
find b.

4. Find the coefficient of x^2 in

$$\sqrt{\frac{1-x}{1+x}}, \qquad |x| < 1.$$

5. Find the first three non-zero terms of

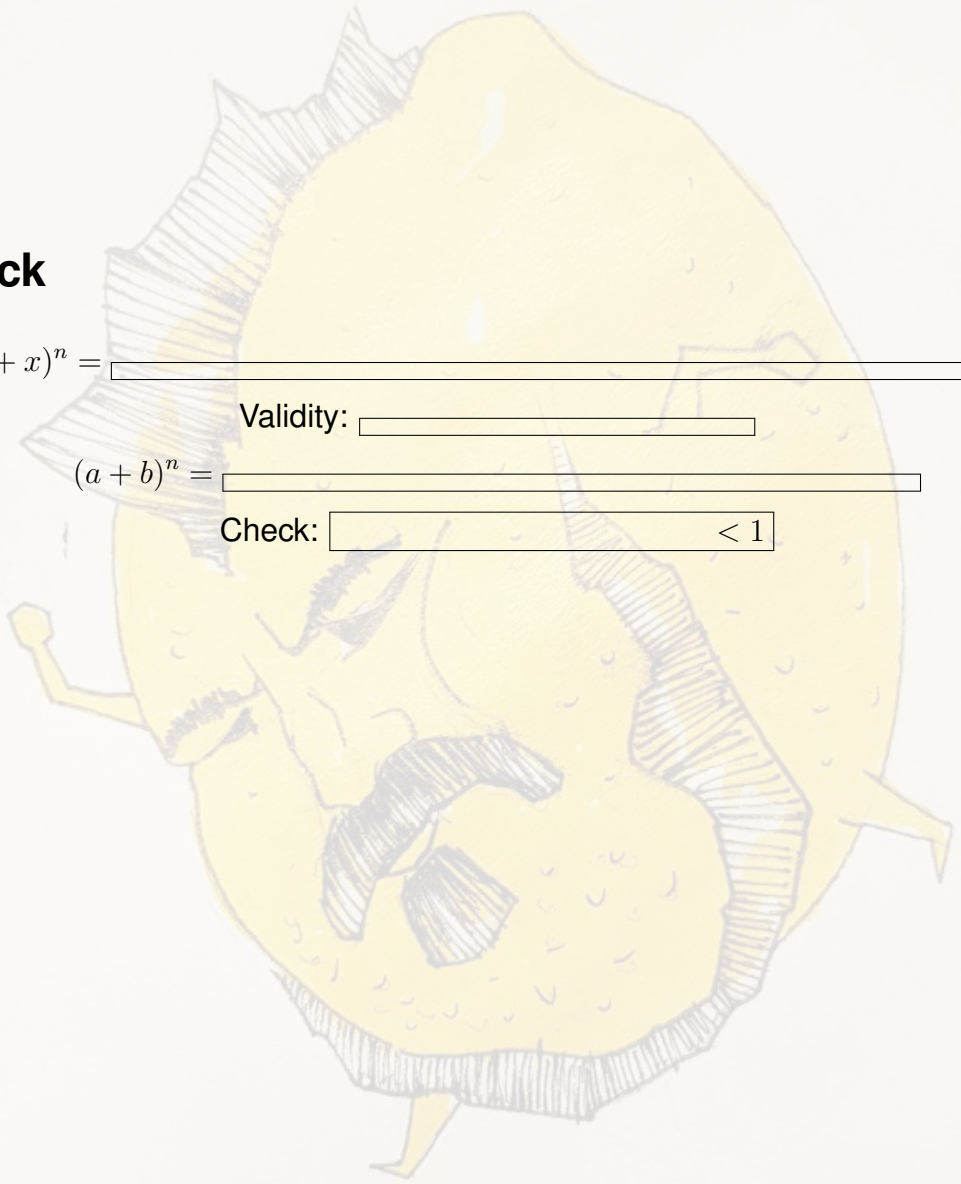
$$\sqrt[3]{\frac{1+2x}{1-x}}.$$



6. Given

$$\frac{x^2}{(1+cx)^2} = x^2 - 2cx^3 + 6x^4 + \cdots,$$

find c.



Final Check

$(1+x)^n =$

Validity:

$(a+b)^n =$

Check: < 1

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Mathematics made clear.